

## Dynamic Relatedness Analysis of Three Stock Market Return Volatility with a Factor of U.S. stock market: Empirical Study of Hong Kong, Japan, and Singapore Countries

<sup>1</sup>Wann-Jyi Horng, <sup>2</sup>Jui-Chen Chang

<sup>1</sup>, *First Author* Department of Hospital and Health Care Administration  
Chia Nan University of Pharmacy & Science, Tainan, Taiwan,  
hwj7902@mail.chna.edu.tw

<sup>\*2</sup>, *Corresponding Author* Department of Finance & Institute of Financial Management  
Nanhua University, Chia-Yi, Taiwan, juichen@mail.nhu.edu.tw

### Abstract

*This paper studies the associations among and the model construction of Hong Kong, Japan, and Singapore's stock markets. In this paper we construct a dynamic conditional correlation (DCC) and a trivariate IGARCH (1, 1) model to evaluate the associations, and find that there does not exist an asymmetrical effect among the three stock markets with a factor of U.S. stock market. The result of empirical correlation analyses also shows that Japan's stock market returns positively affect the Hong Kong and Singapore stock market returns, and the volatility of the three stock market returns interact with one another. Furthermore, the time lags of Hong Kong stock market returns do not affect the returns of the Japanese and Singapore stock markets. The variation risk of the Japan stock market returns' volatility affects the variation risks of Hong Kong and Singapore stock market returns. Empirical results also show that both good and bad news will actually affect the variation risks of stock market returns. Therefore, based on the viewpoint of DCC, the explanatory ability of the trivariate IGARCH(1, 1) model is better than the model of the trivariate GARCH with a constant conditional correlation. The evidence suggests that stock market investors or international fund managers must evaluate the variation risk and relationships of the stock market returns' volatility.*

**Keywords:** Stock Market Returns, Asymmetrical Effect, Trivariate IGARCH Model, DCC

### 1. Introduction

We know that Hong Kong already established sound economic regulations when she was under the control of the U.K., and currently Hong Kong is an important economic and trade area for mainland China. According to the statistics of the Securities and Futures Commission (SFC), at the end of December 2005, the Hong Kong exchange market's turnover was up to US\$1,055 billion, the eighth highest in the world and the second highest in Asia. Hong Kong undoubtedly also plays a very important role in the global economic and financial system. We also know that the economic growth rate of Singapore, one of the Four Asian Tigers, was 7.9% in 2006, and is forecast to expand at a rate of 5.5-7.5% in the future. Moreover, Singapore is the main financial center of Asia, and her foreign exchange market is the fourth largest trading market in the world. In addition, Hong Kong and Singapore have a close relationship with Japan, and the three stock markets are the most important financial markets in Asia. How these three stock markets impact one another certainly merits further discussion. Besides, we also consider the factor of the U.S. stock market in this paper.

In the financial time series non-linear research literature, Engle (1982) proposed the autoregressive conditionally heteroskedasticity (ARCH) model, and Bollerslev (1986) presented the generalization autoregressive conditionally heteroskedasticity (GARCH) model. These two models can find financial properties when the conditional variance is not a fixed parameter. Nelson (1990) considered fluctuations in stock prices and found that they have both positive and negative correlations with future stock price volatility. The GARCH model supposes a settled time conditional variance for

the preceding issue of conditional variance and an error term square function. Therefore, the error term's positive and negative values do not respond to its influence on the conditional variance equation. The conditional variance only changes when the error term's value changes, and cannot go along with the error term's positive and negative changes. To correct this flaw, Nelson (1991) presented an exponential GARCH model, and Glosten, Jaganathan, and Runkle (1993) developed a GJR-GARCH model. These are termed asymmetric GARCH models. There have been many research studies on the asymmetric problems, such as Horng and Kuan (2009), Horng and Lee (2008), Poon and Fung (2000), Campell and Hentschel (1992), Koutmos and Booth (1995), and Koutmos (1996). These studies expanded the research methods used in the area of return volatility of stock markets. For statements on the multivariate GARCH model was proposed, scholars such as Yang and Doong (2004), Yang (2005), Granger, Hung and Yang (2000) and Bollerslev (1990) developed the bivariate GARCH model.

The main goal of this paper is to discuss the associations between Hong Kong, Japan, and Singapore's stock return volatility. The paper constructs a trivariate GARCH theoretical model and examines whether or not there is an asymmetrical influence among the three stock markets. And we will also further discuss the influence of the U.S. stock market on the three stock markets. The organization of this paper is as follows. Section 2 describes the data characteristics of the three stock markets. Section 3 introduces the proposed model and model checking. Section 4 presents the asymmetrical test of the proposed model, and the last section presents conclusions.

## 2. Data characteristics

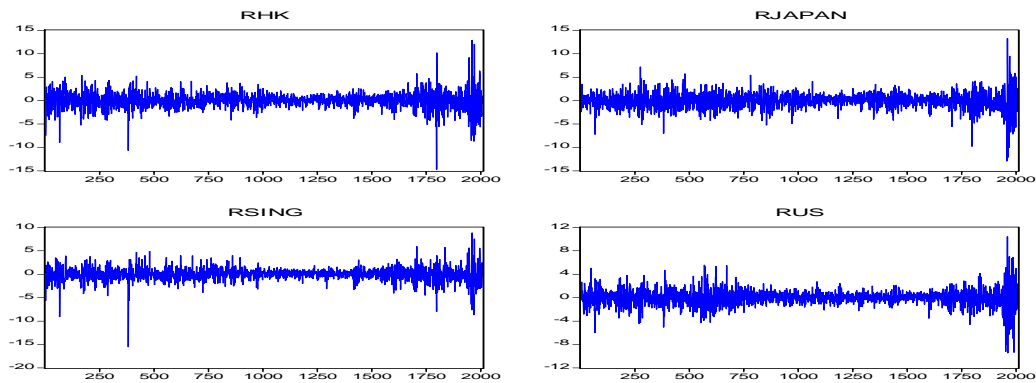
### 2.1 Data sources

This study uses the Hong Kong Hang Seng index, the Singapore Straits Times index, the Japan Tokyo NK-225 index and S&P500 index as the sample. The data period is from January 2000 to December 2008, and uses the date data for all stock price indices, with the data are obtained from the Taiwan Economic Journal (TEJ), a database in Taiwan. For treat the data processing, we have already considered the stock markets' common trading days, so the sample size is 2006 for all three stock markets, respectively.

### 2.2 Returns calculation and trend of charts

To compute the stock price return rates, this paper adopts the natural logarithm of the stock price index for every stock market sample ( $HK_t, SING_t, JAPAN_t, US_t$ ) with one step difference and then multiplied by 100, i.e. for the Hong Kong stock market, the stock price return rates are  $RHK_t = 100 * [\ln(HK_t / HK_{t-1})]$ . For the Singapore stock market, the stock price return rates are  $RSING_t = 100 * [\ln(SING_t / SING_{t-1})]$ . For the Japan stock market, the stock price return rates are  $RJAPAN_t = 100 * [\ln(JAPAN_t / JAPAN_{t-1})]$ . For the U.S. stock market, the stock price return rates are  $RUS_t = 100 * [\ln(US_t / US_{t-1})]$ . Figure 1 shows the trend charts of Hong Kong, Japan, Singapore and U.S. tock price indices during the sample period. Figure 2 shows those of the Hong Kong, Japan, Singapore and U.S. stock price return rates.

From Figure 1, we also know that the clustering phenomenon is present in the volatility of these three stock market returns. We may also know that Hong Kong, Japan and Singapore's stock markets have a certain relationship in their return volatility processes. And U.S. stock market returns may affect the return volatility of these three stock market returns. This also shows that there are mutual relationships in stock returns among these three markets. This is one of main motivations for discussing the relationships between Hong Kong, Japan and Singapore's stock price returns with a factor of U.S. stock market.



**Figure 1.** Trend charts of return rates of Hong Kong, Japan, Singapore and U.S. stock price indices.

### 2.3 Statistics

Table 1 shows some basic statistical analyses: mean value, standard deviation, kurtosis coefficient, skewed coefficient, and normal distribution examination. As shown in Table 1, the average return rate of the Hong Kong stock price index is -0.0091, the average return rate of the Singapore stock price index is -0.0178, and the average return rate of the Japanese stock price index is -0.0381. The variation risk of the Hong Kong stock price return rate is 1.7078, the variation risk of the Singapore stock price return rate is 1.4165, and the variation risk of the Japanese stock price return rate is 1.6989, making the variation risk of the Hong Kong's stock price return rate the highest. From the normal distribution test of Jarque-Bera, we know that the three stock price return rates are not a normal distribution. Note that the kurtosis coefficients of three sequences are larger than 3, and this demonstrates that the study data reflect the heavy tail distribution phenomenon. We know that as the sample is large enough, the analytical result of the heavy tail distribution approaches that of the normal distribution. The stock price return rates for the Hong Kong, Japan, Singapore and U.S. markets show a stationary state sequence, as shown below in Table 2.

**Table 1.** Statistical data

Statistic	RHK	RJAPAN	RSING	RUS
Mean	-0.009065	-0.038059	-0.017802	-0.022537
S-D	1.707781	1.698881	1.416549	1.421452
Skewed	-0.274595	-0.463023	-0.942781	-0.283211
Kurtosis	11.89092	10.76508	14.70459	10.26465
J-B N	6629.038 ***	5108.904 ***	11742.02 ***	4435.732 ***
(p-value)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
sample	2005	2005	2005	2005

Notes: (1) S-D denotes the standard deviation of data. (2) J-B N denotes the Jarque-Bera normal distribution test. (3) p-value  $< \alpha$  denotes significance ( $\alpha = 1\%$ ,  $\alpha = 5\%$ ,  $\alpha = 10\%$ ).

\*\*\* denotes significance at the level 1%.

### 2.4 Unit root test

In order to find a suitable model, one may first determine the stability of the time series data, as well

as avoid the non-stationary state of the time series sequences and reduce the mistake of the empirical result. To do so, this paper further uses the unit root test of Dickey-Fuller (1979 and 1981, ADF) and KSS (Kapetanios et al., 2003) to determine the stability of the time series data. The ADF and KSS examination results are listed in Table 2, and show that the Hong Kong, Japan and Singapore's stock price return rates do not have the unit root characteristic. This indicates that the time series data is stable, under  $\alpha = 1\%$  significance level. Therefore, we can further analyze the time series data of the three stock markets.

**Table 2.** ADF and KSS-Unit root test of the data

<b>ADF</b>	<b>RHK</b>	<b>RJAPAN</b>	<b>RSING</b>	<b>RUS</b>
Statistic	-10.0771 ***	-10.4063 ***	-11.2349 ***	-11.6874 ***
C-V	-3.9627	-3.4121	-3.1280	
(S-L)	( $\alpha = 1\%$ )	( $\alpha = 5\%$ )	( $\alpha = 10\%$ )	
<b>KSS</b>	<b>RHK</b>	<b>RJAPAN</b>	<b>RSING</b>	<b>RUS</b>
Statistic	-24.9502 ***	-21.0057 ***	-16.4494 ***	-26.4698 ***
C-V	-2.82	-2.22	-1.92	
(S-L)	( $\alpha = 1\%$ )	( $\alpha = 5\%$ )	( $\alpha = 10\%$ )	

Notes : (1) C-V denotes the critical value and S-L denotes significance level.

(2) \*\*\* denotes significance at the level 1%.

## 2.5 Co-integration test

From the co-integration test of Johansen (1991), we know that the statistics of  $\lambda_{\max}$  and the Trace are not significant under the level of 5% in Table 3. This demonstrates that these four stock markets do not have co-integrated relations. In Table 4, the unconditional correlation matrix of Hong Kong, Japan, Singapore and U.S. stock market returns shows correlation. Although the Hong Kong, Japan and Singapore stock markets do not have long-term co-integrated relations, these three markets actually do affect one another. Therefore, we investigate further to understand the relations between the three stock markets with a factor of U.S. stock market.

**Table 3.** Johansen's co-integration test (the lag of VAR is 4)

<b>Null</b> ( $H_0$ )	$\lambda_{\max}$	<b>C-V</b> ( $\alpha = 5\%$ )	<b>Trace</b>	<b>C-V</b> ( $\alpha = 5\%$ )
None	22.4846	27.5843	45.1709	47.8561
At most 1	15.0216	21.1316	22.6863	29.7971
At most 2	4.7331	14.2646	7.6648	15.4947
At most 3	2.9317	3.8415	2.9317	3.8415

Notes : (1) C-V denotes the critical value. (2) The lag of VAR is selected by the AIC rule (Akaike, 1973).

**Table 4.** Unconditional correlation matrix

Coefficient	RHK	RJAPAN	RSING	RUS
RHK	1	0.6391	0.7390	0.2401
RJAPAN	0.6391	1	0.6059	0.1882
RSING	0.7390	0.6059	1	0.2660
RUS	0.2401	0.1882	0.2660	1

## 2.6 ARCH effect test

The ARCH effect test is used to determine stock return volatility and whether the conditional heteroskedasticity phenomenon is present. We also use the Ljung-Box (1978) test method, the Lagrange Multiplier (LM) test method of Engle (1982), and the F distribution test method of Tsay (2004) to further confirm the variance of the residual error sequence and whether or not there is an ARCH effect. If there is an ARCH effect, we use the GARCH model to match it suitably. The ARCH effect test takes the residual error square of the past  $q$  periods to carry out regression analysis. The ARCH effect test is based on the AR model in Table 8. Its mathematical form is:

$$\hat{a}_t^2 = d_0 + d_1 \hat{a}_{t-1}^2 + \dots + d_q \hat{a}_{t-q}^2 + v_t \quad (1)$$

We test the null hypotheses  $H_0 : d_1 = d_2 = \dots = d_q = 0$  by (1). When  $H_0$  is rejected, it means that the ARCH effect does exist, and so we can use the GARCH model to fit it.

We implement the LM, F, and Ljung-Box (L-B) test methods to examine the stock price date returns and to determine whether or not there is a conditional heteroskedasticity phenomenon. The results of the ARCH effect test for the three stock markets are listed in Table 5. The results show that the analytical model of the three stock price return rates has a significant statistical value under the level  $\alpha = 5\%$  and a conditional heteroskedasticity phenomenon exists. This suggests that it is a suitable match, and the GARCH model could be used to analyze data.

**Table 5.** ARCH effect test (lag=30)

	Engle LM test		Tsay F test		L-B test	
					$LB^2(1)$	$LB^2(3)$
Hong Kong	Statistic	437.9637 ***	Statistic	18.4667 ***	8.7867 ***	12.8034 ***
	(p-value)	(0.0000)	(p-value)	(0.0000)	(0.0000)	(0.0000)
Japan	Statistic	580.1142 ***	Statistic	26.9549 ***	5.2305 ***	15.4304 ***
	(p-value)	(0.0000)	(p-value)	(0.0000)	(0.0000)	(0.0000)
Singapore	Statistic	123.5555 ***	Statistic	4.3245 ***	4.5332 ***	3.2463 ***
	(p-value)	(0.0000)	(p-value)	(0.0000)	(0.0000)	(0.0012)

Notes:  $p\text{-value} < \alpha$  denotes significance ( $\alpha = 1\%$ ,  $\alpha = 5\%$ ,  $\alpha = 10\%$ ). \* denotes significance at the level 10%, \*\* denotes significance at the level 5%, and \*\*\* denotes significance at the level 1%.

### 3. Proposed model and model checking

#### 3.1 Trivariate GARCH model and parameter estimation

Follow the ideas of Shih, Hsiao and Chen (2008) and Tsay (2004), this section uses the trivariate GARCH model- i.e. (2)-(14) types to analyze the relatedness of Hong Kong, Japan and Singapore's stock price return volatilities with a factor of U.S. stock market. The parameter estimation first considers a general model and is based on the estimated results. We then delete some explanatory variables that are not significant. Finally, we obtain a simplified model for the analysis of the relatedness of Hong Kong, Japan and Singapore's stock price return volatilities. The empirical results show that Hong Kong, Japan and Singapore's stock price return volatility may be constructed on the trivariate IGARCH(1, 1) model. Its estimated results are in Table 6. Proposed model of the trivariate IGARCH(1, 1) model is given as follows:

$$RHK_t = \phi_{21}RJAPAN_{t-1} + \phi_{31}RSING_{t-1} + \phi_{41}RUS_{t-1} + a_{1,t}, \quad (2)$$

$$RJAPAN_t = \varphi_{11}RJAPAN_{t-1} + \varphi_{31}RSING_{t-1} + \varphi_{41}RUS_{t-1} + a_{2,t}, \quad (3)$$

$$RSING_t = \psi_{11}RJAPAN_{t-1} + \psi_{31}RSING_{t-1} + \psi_{41}RUS_{t-1} + a_{3,t}, \quad (4)$$

$$h_{11,t} = \alpha_{10} + \alpha_{11}a_{1,t-1}^2 + \beta_{11}h_{11,t-1} + \eta_1RUS_{t-1}^2, \quad (5)$$

$$h_{22,t} = \alpha_{20} + \alpha_{22}a_{2,t-2}^2 + \beta_{21}h_{22,t-1} + \eta_2RUS_{t-1}^2, \quad (6)$$

$$h_{33,t} = \alpha_{30} + \alpha_{31}a_{3,t-1}^2 + \beta_{31}h_{33,t-1} + \eta_3RUS_{t-1}^2, \quad (7)$$

$$q_{12,t} = c_0 + c_1\rho_{12,t-1} + c_2a_{1,t-1}a_{2,t-1} / \sqrt{h_{11,t-1}h_{22,t-1}}, \quad (8)$$

$$q_{13,t} = d_0 + d_1\rho_{13,t-1} + d_2a_{1,t-1}a_{3,t-1} / \sqrt{h_{11,t-1}h_{33,t-1}}, \quad (9)$$

$$q_{23,t} = e_0 + e_1\rho_{23,t-1} + e_2a_{2,t-1}a_{3,t-1} / \sqrt{h_{22,t-1}h_{33,t-1}}, \quad (10)$$

$$\rho_{12,t} = \exp(q_{12,t}) / (\exp(q_{12,t}) + 1), \quad (11)$$

$$\rho_{13,t} = \exp(q_{13,t}) / (\exp(q_{13,t}) + 1), \quad (12)$$

$$\rho_{23,t} = \exp(q_{23,t}) / (\exp(q_{23,t}) + 1), \quad (13)$$

$$h_{12,t} = \rho_{12} \sqrt{h_{11,t}} \sqrt{h_{22,t}}, \quad h_{13,t} = \rho_{13} \sqrt{h_{11,t}} \sqrt{h_{33,t}}, \quad h_{23,t} = \rho_{23} \sqrt{h_{22,t}} \sqrt{h_{33,t}}, \quad (14)$$

$\bar{a}'_t = (a_{1,t}, a_{2,t}, a_{3,t})$  obeys the trivariate normal distribution- namely,  $N(\bar{0}, H_t)$ , among  $\bar{0}' = (0,0,0)$  and  $H_t$  is an  $3 \times 3$  covariance matrix of  $\bar{a}_t$  with  $h_{12,t} = h_{21,t}$ ,  $h_{13,t} = h_{31,t}$ ,  $h_{23,t} = h_{32,t}$ . The probability density function of  $\bar{a}_t$  can refer the book of Tsay (2004). And  $\rho_{12,t}$  is the dynamic conditional correlation (DCC) coefficient of  $a_{1,t}$  and  $a_{2,t}$ ,  $\rho_{13,t}$  is the DCC coefficient of  $a_{1,t}$  and  $a_{3,t}$ , and  $\rho_{23,t}$  is the DCC coefficient of  $a_{2,t}$  and  $a_{3,t}$ . In this paper, we use the normal distribution for the stochastic error term, and also use the maximum likelihood algorithm method of BHHH (Berndt et. al., 1974) to estimate the parameters of the proposed model.

With the estimated results of the trivariate IGARCH(1, 1) model in Table 6, we use a P-value to test whether the estimated value of the parameters' coefficient is significant. During the sample period, Hong Kong stock price returns were not affected by the constant term and time lags of the Hong Kong

stock returns. The Hong Kong stock price return receives the time lags' influence of the Japan and Singapore stock price returns with the time lags equals 1. And the Hong Kong stock price return receives the time lags' influence of the U.S. stock price returns with the time lags equals 1. The Japan stock price return is not affected by the constant term and the Hong Kong stock price returns' time lags. Japanese stock price returns receives the influence of Japan and Singapore stock price returns with the time lags equals 1. And the Japanese stock price return receives the time lags' influence of the U.S. stock price returns with the time lags equals 1. Similarly, Singapore stock price returns is also not affected by the constant term and the Hong Kong stock price returns' time lags. The Singapore stock price return receives the affect of the Japan and U.S. stock price returns with the time lags equals 1. On the other hand, the correlation coefficient value of Hong Kong and Japanese stock price return volatilities is significant ( $\bar{\rho}_{12}=0.5304$ ). This result means that Japanese stock price returns' volatility has a positive influence on Hong Kong stock price returns' volatility, and they have precisely synchronized mutual influence. When the variation risk of Japanese stock price returns increases, investors' risk in Hong Kong stock price market can increase. Likewise, when the variation risk of the Japan stock price return decreases, investors' risk in the Hong Kong stock price market can also decrease. Similarly, the correlation coefficient value of Hong Kong and the Singapore stock price return volatilities is significant ( $\bar{\rho}_{13}=0.6373$ ), and the two stock price markets have a high degree of relatedness. This result also shows that Hong Kong stock price returns' volatility has a positive influence over Singapore stock price returns' volatility. The correlation coefficient value of the Singapore and the Japan stock price return volatilities is also significant ( $\bar{\rho}_{23}=0.5313$ ). This result also shows that Japanese stock price returns' volatility has positive influence on Singapore stock price returns' volatility, and the two stock price markets also have a high degree of relatedness.

The observed conditional variance equation of the estimated coefficient, under the 1% significance level, demonstrates that all the conditional variance estimated coefficients in Table 6 are significant. The empirical results show that the previous one periods' residual error square item and the previous period's conditional variance will affect Hong Kong, Singapore, and Japanese stock price return rate volatilities and can also produce different variation risks. In Table 6, we have the variation risk for the Singapore's stock price market is the lowest ( $\beta_{31} = 0.7842$ ). These three stock markets have fixed variation risks. The fixed variation risk for Hong Kong stock market prices is the lowest ( $\alpha_{10} = 0.0395$ ). The square item of the U.S. stock market returns also affects the variation risk of these three stock markets. An impact affect for Hong Kong stock market returns is the lowest ( $\eta_1 = 0.0429$ ). Moreover,  $\alpha_{11} + \beta_{11} + \eta_1 = 1$ ,  $\alpha_{22} + \beta_{21} + \eta_2 = 1$ , and  $\alpha_{31} + \beta_{31} + \eta_3 = 1$  conforms to the parameters of the IGARCH model's conditional supposition.

**Table 6.** Parameter estimation of the trivariate IGARCH(1, 1) model

Parameter	$\phi_{21}$	$\phi_{31}$	$\phi_{41}$	$\phi_{11}$	$\phi_{31}$
Coefficient	-0.1020 <sup>***</sup>	0.0564 <sup>**</sup>	0.4424 <sup>***</sup>	-0.0813 <sup>***</sup>	0.0588 <sup>**</sup>
(p-value)	(0.0000)	(0.0461)	(0.0000)	(0.0053)	(0.0439)
Parameter	$\phi_{41}$	$\psi_{11}$	$\psi_{31}$	$\psi_{41}$	$\alpha_{10}$
Coefficient	0.4850 <sup>***</sup>	-0.0416 <sup>**</sup>	-0.0179	0.3074 <sup>***</sup>	0.0395 <sup>***</sup>
(p-value)	(0.0000)	(0.0473)	(0.5475)	(0.0000)	(0.0000)
Parameter	$\alpha_{11}$	$\beta_{11}$	$\eta_1$	$\alpha_{20}$	$\alpha_{21}$
Coefficient	0.1045 <sup>***</sup>	0.8526 <sup>***</sup>	0.0429 <sup>***</sup>	0.0749 <sup>***</sup>	0.1255 <sup>***</sup>
(p-value)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Parameter	$\beta_{21}$	$\eta_2$	$\alpha_{30}$	$\alpha_{31}$	$\beta_{31}$
Coefficient	0.8041 <sup>***</sup>	0.0704 <sup>***</sup>	0.0453 <sup>***</sup>	0.1062 <sup>***</sup>	0.7842 <sup>***</sup>
(p-value)	(0.0051)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Parameter	$\eta_3$	$c_0$	$c_1$	$c_2$	$d_0$
Coefficient	0.1096 <sup>***</sup>	-1.5370 <sup>***</sup>	2.9819 <sup>***</sup>	0.1672 <sup>***</sup>	-1.8843 <sup>***</sup>
(p-value)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Parameter	$d_1$	$d_2$	$e_0$	$e_1$	$e_2$
Coefficient	3.7335 <sup>***</sup>	0.1605 <sup>***</sup>	-1.8387 <sup>***</sup>	3.5851 <sup>***</sup>	0.1341 <sup>***</sup>
(p-value)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Parameter	$\bar{\rho}_{12}$	$\bar{\rho}_{13}$	$\bar{\rho}_{23}$	min $\rho_{12}$	max $\rho_{12}$
Coefficient	0.5304 <sup>***</sup>	0.6373 <sup>***</sup>	0.5313 <sup>***</sup>	0.3120	1.0000
(p-value)	(0.0000)	(0.0000)	(0.0000)		
Parameter	min $\rho_{13}$	max $\rho_{13}$	min $\rho_{23}$	max $\rho_{23}$	
Coefficient	0.3994	1.0000	0.2211	1.0000	
(p-value)					

Notes: (1) p-value  $< \alpha$  denotes significance ( $\alpha = 1\%$ ,  $\alpha = 5\%$ ,  $\alpha = 10\%$ ).

(2) \* denotes significance at the level 10%, \*\* denotes significance at the level 5%, and

\*\*\* denotes significance at the level 1%.

### 3.2 Model checking of the standard residual error for the trivariate IGARCH model

The trivariate IGARCH model will be appropriate for examining the standard residual error and a standard residual error square series by the test method of Ljung-Box and determining whether they still have auto-correlation. This is done by the standard residual error Q test of  $LB(5)$  to  $LB(25)$  with a P-value and the standard residual error square series Q test of  $LB^2(5)$  to  $LB^2(25)$  with a P-value in Table 7. The diagnosis finds that the trivariate GARCH(1, 2) model already does not have an auto-correlation with the standard residual error. This is also shown by Table 8. The trivariate IGARCH(1, 1) model already does not have an ARCH effect on the standard residual error square



series. Therefore, the proposed model is suitably matched and is appropriate for these purposes.

**Table 7.** Q test of the standard residual error and its squared series

<b>Hong Kong</b>	<i>LB</i> (5)	<i>LB</i> (15)	<i>LB</i> (25)	<i>LB</i> <sup>2</sup> (5)	<i>LB</i> <sup>2</sup> (15)	<i>LB</i> <sup>2</sup> (25)
Q statistic	4.8935	17.5078	19.0266	3.9208	8.2622	15.1993
(p-value)	(0.4290)	(0.2894)	(0.7958)	(0.5609)	(0.9128)	(0.9663)
<b>Japan</b>	<i>LB</i> (5)	<i>LB</i> (15)	<i>LB</i> (25)	<i>LB</i> <sup>2</sup> (5)	<i>LB</i> <sup>2</sup> (15)	<i>LB</i> <sup>2</sup> (25)
Q statistic	3.8963	9.3743	15.3638	6.3655	11.7229	19.9773
(p-value)	(0.5644)	(0.8571)	(0.9325)	(0.2723)	(0.6999)	(0.7480)
<b>Singapore</b>	<i>LB</i> (5)	<i>LB</i> (15)	<i>LB</i> (25)	<i>LB</i> <sup>2</sup> (5)	<i>LB</i> <sup>2</sup> (15)	<i>LB</i> <sup>2</sup> (25)
Q statistic	2.7055	13.8492	20.0693	0.4222	1.1603	1.9342
(p-value)	(0.7453)	(0.5370)	(0.7432)	(0.9947)	(1.0000)	(1.0000)

Notes:  $p\text{-value} < \alpha$  denotes significance ( $\alpha = 1\%$ ,  $\alpha = 5\%$ ,  $\alpha = 10\%$ ). \* denotes significance at the level 10%, \*\* denotes significance at the level 5%, and \*\*\* denotes significance at the level 1%.

**Table 8.** L-B test-ARCH effect test of the standard residual error

<b>Hong Kong</b>	<i>LB</i> <sup>2</sup> (10)	<i>LB</i> <sup>2</sup> (20)	<i>LB</i> <sup>2</sup> (30)	F test	
Q statistic	-1.1893	-0.3774	0.2839	Statistic	0.4946
(p-value)	(0.2345)	(0.7059)	(0.7765)	(p-value)	(0.9904)
<b>Japan</b>	<i>LB</i> <sup>2</sup> (10)	<i>LB</i> <sup>2</sup> (20)	<i>LB</i> <sup>2</sup> (30)	F test	
Q statistic	1.3827	0.7203	0.1604	Statistic	0.7518
(p-value)	(0.1669)	(0.4714)	(0.8726)	(p-value)	(0.8320)
<b>Singapore</b>	<i>LB</i> <sup>2</sup> (10)	<i>LB</i> <sup>2</sup> (20)	<i>LB</i> <sup>2</sup> (30)	F test	
Q statistic	-0.3402	0.0151	-0.0788	Statistic	0.0724
(p-value)	(0.7337)	(0.9880)	(0.9372)	(p-value)	(1.0000)

Notes:  $p\text{-value} < \alpha$  denotes significance ( $\alpha = 1\%$ ,  $\alpha = 5\%$ ,  $\alpha = 10\%$ ).

\* denotes significance at the level 10%, \*\* denotes significance at the level 5%, and

\*\*\* denotes significance at the level 1%.

#### 4. Asymmetric test of the trivariate IGARCH(1, 1) model

Because of the parameter estimation and the standard residual error diagnosis in the above IGARCH(1, 1) model, the examination can only determine if the model matches the suitable quality, and cannot determine whether the model has an asymmetrical phenomenon. Therefore, Engle and Ng (1993) developed a diagnosis test to examine whether or not the model has asymmetrical risk, and we use this diagnostic test to carry out the examination.

Engle and Ng (1993) considered that by observing the variables' past value, it is possible to forecast the standardized residual error square  $(a_t / \sigma_t)^2$ ,  $\sigma_t = (h_t)^{1/2}$ . However, if there is no forecast pattern of the variables' past value, then the expression model may be set up incorrectly.

Therefore, there are four examination methods for the model hypotheses:

(1) Sign bias test

$$(a_t / \sigma_t)^2 = b_0 + b_1 S_{t-1}^- + e_t, \quad (15)$$

(2) Negative size bias test

$$(a_t / \sigma_t)^2 = b_0 + b_1 S_{t-1}^- (a_{t-1} / \sigma_{t-1}) + e_t, \quad (16)$$

(3) Positive size bias test

$$(a_t / \sigma_t)^2 = b_0 + b_1 (1 - S_{t-1}^-) (a_{t-1} / \sigma_{t-1}) + e_t, \quad (17)$$

(4) Joint test

$$(a_t / \sigma_t)^2 = b_0 + b_1 S_{t-1}^- + b_2 S_{t-1}^- (a_{t-1} / \sigma_{t-1}) + b_3 (1 - S_{t-1}^-) (a_{t-1} / \sigma_{t-1}) + e_t, \quad (18)$$

where  $S_{t-1}^-$  is the dummy variable; as  $a_t \leq 0$ , then  $S_t^- = 1$ ; as  $a_t > 0$ , then  $S_t^- = 0$ .

After the above-mentioned results, Table 9 asymmetrically examines the results for Hong Kong stock market prices, indicating that: (a) The sign bias test does not reveal ( $\alpha = 10\%$ ). (b) The negative size bias test does not reveal ( $\alpha = 10\%$ ). (c) The positive size bias test reveals ( $\alpha = 10\%$ ). (d) The joint test does not reveal ( $\alpha = 10\%$ ). Table 9 asymmetrically examines the results for Japanese stock market prices, indicating that: (a) The sign bias test does not reveal ( $\alpha = 10\%$ ). (b) The negative size bias test does not reveal ( $\alpha = 10\%$ ). (c) The positive size bias test does not reveal ( $\alpha = 10\%$ ). (d) The joint test does not reveal ( $\alpha = 10\%$ ). Table 9 asymmetrically examines the results for Singapore stock market prices, indicating that: (a) The sign bias test does not reveal ( $\alpha = 10\%$ ). (b) The negative size bias test does not reveal ( $\alpha = 10\%$ ). (c) The positive size bias test does not reveal ( $\alpha = 10\%$ ). (d) The joint test does not reveal ( $\alpha = 10\%$ ). From the joint test, it is shown that an asymmetrical phenomenon do not exist among the of the Hong Kong, Japan, and Singapore's stock market prices during the sample period.

**Table 9.** Asymmetric test of the trivariate IGARCH model

<b>Hong Kong</b>	<b>Sign bias test</b>	<b>Negative size Bias test</b>	<b>Positive size Bias test</b>	<b>Joint test</b>
F statistic	0.3659	1.4101	3.5963*	1.3970
(p-value)	(0.5453)	(0.2352)	(0.0581)	(0.2419)
<b>Japan</b>	<b>Sign bias test</b>	<b>Negative size Bias test</b>	<b>Positive size Bias test</b>	<b>Joint test</b>
F statistic	1.5588	0.6348	1.4009	1.5861
(p-value)	(0.2120)	(0.4257)	(0.2367)	(0.1907)
<b>Singapore</b>	<b>Sign bias test</b>	<b>Negative size Bias test</b>	<b>Positive size Bias test</b>	<b>Joint test</b>
F statistic	1.9875	1.3320	0.0131	1.1960
(p-value)	(0.1588)	(0.2486)	(0.9089)	(0.3098)

Notes: p-value  $< \alpha$  denotes significance ( $\alpha = 1\%$ ,  $\alpha = 5\%$ ,  $\alpha = 10\%$ ). \* denotes significance at the level 10%, \*\* denotes significance at the level 5%, and \*\*\* denotes significance at the level 1%.

## 5. Conclusions

There are many factors that may influence stock prices, including overall economic agents, overall currency supplies, interest rates, prices, and inflation rates. Each factor can influence stock price returns. This study has discussed three stock market return volatilities that influence the Hong Kong, Japan and Singapore's markets. The empirical results show that Hong Kong, Japan and Singapore's stock price return volatilities may be constructed in the trivariate normal distribution and the trivariate IGARCH(1, 1) model. This model also passes a standard residual error and ARCH effect tests. This demonstrates that the trivariate IGARCH(1, 1) model is a more appropriate fit. The empirical results also show that the correlation coefficient value ( $\bar{\rho}_{12}=0.5304$ ) for Hong Kong and Japanese stock market returns has positive relations, the correlation coefficient value ( $\bar{\rho}_{13}=0.6373$ ) for Hong Kong and Singapore stock market returns has positive relations, and the correlation coefficient value ( $\bar{\rho}_{23}=0.5313$ ) for the Japan and Singapore stock market returns also has positive relations. This result demonstrates that Japanese stock return volatility affects Hong Kong and Singapore stock returns' variation risk, and Hong Kong stock return volatility does not affect Japanese stock returns' variation risk. The empirical results also show that Hong Kong, Japan and Singapore's stock market returns' volatilities do not have the asymmetrical phenomenon during the sample period. However, if investors can clearly understand the process of stock market return volatilities, it will be easier for them to evaluate the three stock markets and their investment returns and to determine an investment strategy.

We also know that there are many theories and models for the returns and the volatility properties of financial commodities. This research uses only the trivariate asymmetric GARCH model to discuss the three stock markets of Hong Kong, Japan and Singapore. For future research, we suggest that other asymmetries of the multivariate GARCH model or other models can be used for further analysis.

## References

- [1] Akaike H., "Information theory and an extension of the maximum likelihood principle, In 2nd, International Symposium on Information Theory", edited by B.N. Petrov and F.C. Budapest: Akademiai Kiado, pp.267-281, 1973.
- [2] Bollerslev T., "Generalized autoregressive conditional hetroskedasticity", Journal of Econometrics, vol.31, pp.307-327, 1986.
- [3] Bollerslev T., "Modeling the coherence in short-run nominal exchange rates: a multivariate generalized ARCH model", Review of Economics and Statistics, vol.72, pp.498-505, 1990.
- [4] Berndt E.K., Hall B.H., Hall R.E., Hausman J.A., "Estimation and inference in nonlinear structural models", Annals of Economic and Social Measurement, vol.4, pp.653-665, 1974.
- [5] Campell J.Y., Hentschel L., "No news is good news: an asymmetric model of changing volatility in stock returns", Journal of Financial Economics, vol.31, pp.281-318, 1992.
- [6] Dickey D.A., Fuller W.A., "Distribution of estimates for autoregressive time series with a unit root", Journal of the American Statistical Association, vol.74, pp.427-431, 1979.
- [7] Dickey D.A., Fuller W.A., "Likelihood ratio statistics for autoregressive time series with a unit root", Econometrica, vol.49, pp.1057-1072, 1981.
- [8] Engle R.F., "Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom Inflation", Econometrica, vol.50, pp.987-1007, 1982.
- [9] Engle R.F., Ng V.K., "Measuring and testing the impact of news on volatility", Journal of Finance, vol.48, no.5, pp.1749-1777, 1993.
- [10] Granger C.W., Hung J.B., Yang C.W., "A bivariate causality between stock prices and exchange rates: evidence from recent Asian Flu", The Quarterly Review of Economics and Finance, vol.40, pp.337-354, 2000.
- [11] Glosten L.R., Jagannathan R., Runkle D.E., "On the relation between the expected value and the volatility on the nominal excess returns on stocks", Journal of Finance, vol.48, pp.1779-1801, 1993.
- [12] Horng W.J., Lee J.Y., "An impact of the U.S. and the U.K. return rates' volatility on the stock market returns: an evidence study of Germany's stock market returns", Proceedings IEEE Computer Society, ICCIT2008, vol.2, pp.1159-1163, 2008.
- [13] Horng W.J., Kuan C.M., "A DCC Analysis of Two Exchange Rate Market Returns Volatility with an Japan Dollars Factor: Study of Taiwan and Korea's Exchange Rate Markets", Journal of Convergence Information Technology, Vol. 4, No. 4, pp.7-13, 2009.

- [14] Johansen S., "Estimation and hypothesis testing of cointegration vector in gaussian vector autoregressive models", *Econometrica*, vol.59, pp.1551-1580, 1991.
- [15] Koutmos G., Booth G.G., "Asymmetric volatility transmission in international stock markets", *Journal of International Money and Finance*, vol.14, pp.747-762, 1995.
- [16] Koutmos G., "Modeling the dynamic interdependence of major European stock markets", *Journal of Business Finance and Accounting*, vol.23, pp.975-988, 1996.
- [17] Kapetanios G., Shin Y., Snell A., "Testing for a unit root in the nonlinear STAR framework", *Journal of Econometrics*, vol.112, no.2, pp.359-379, 2003.
- [18] Ljung G.M., Box G.E.P., "On a measure of lack of fit in time series models", *Biometrika*, vol.65, pp.297-303, 1978.
- [19] Nelson D.B., "Stationarity and persistence in the GARCH(1, 1) model", *Econometric Theory*, vol.6, pp.318-334, 1990.
- [20] Nelson D.B., "Conditional heteroskedasticity in asset returns: A new approach", *Econometrica*, vol.59, pp.347-370, 1991.
- [21] Poon W.P.H., Fung H.G., "Red chip or H shares: which China-backed securities process information the fastest?", *Journal of Multinational Financial Management*, vol.10, pp.315-343, 2000.
- [22] Shih M.L., Hsiao S.H., Chen F.S., "The association of China stock index with Japan and US", *Journal of Convergence Information Technology*, vol. 3, no. 2, pp.13-22, 2008.
- [23] Tsay R.S., *Analysis of Financial Time Series*, New York: John Wiley & Sons, Inc., 2004.
- [24] Yang S.Y., Doong S.C., "Price and volatility spillovers between stock prices and exchange rates: empirical evidence from the G-7 countries", *International Journal of Business and Economics*, vol.3, no.2, pp.39-153, 2004.
- [25] Yang S.Y., "A DCC analysis of international stock market correlations: the role of Japan on the Asian Four Tigers", *Applied Financial Economics Letters*, vol.1, no.2, pp.89-93, 2005.